

Enhancing Mathematics Skill and Self-Regulatory Competency through Observation and Emulation

Darshanand Ramdass

*The Graduate School and University Center
City University of New York*

Abstract

This experimental study examined the effects cognitive modeling and social feedback on the mathematics skill and self-regulatory development of 88 fifth-grade students. The students were assigned randomly to one of four conditions: modeling with feedback, modeling without feedback, feedback without modeling, and no modeling or feedback. In the modeling conditions, students observed a model verbalized, demonstrated, and explained procedures to solve fraction problems. In the feedback conditions, students received feedback on the accuracy of their answers and when they made an error the researcher circled it and prompted students to correct them. Multivariate analysis of covariance showed a main effect of modeling, but no effect of feedback. Subsequent univariate analyses of covariance revealed a significant main effect for modeling on each of the dependent measures and a significant cognitive modeling \times feedback interaction on the mathematics posttest, self-efficacy bias, and self-evaluation bias. The implications of the findings are discussed.

Keywords: cognitive modeling, feedback, fifth graders, self-regulation, mathematics

Introduction

Mathematics literacy is a serious problem in United States, many Western, and third-world countries. According to Hetch, Vagi, and Torgeson (2007), students have difficulties with fractions in elementary grades and competency in fraction skills is essential for success in algebra and advanced mathematics courses. Mathematics may be the “new literacy” (Schoenfeld, 1995) because the workforce in the near future will have to handle quantitative skills more skillfully than at present (National Mathematics Advisory Panel, 2008). One widespread goal of education in mathematics at the elementary level is “students’ study and use of numbers should be extended to include larger numbers, fractions, and decimals” (National Council of Teachers of Mathematics, 2000). The Panel (2008) specifically suggests that by Grade 4, students should be proficient in whole number operations (i.e., addition, subtraction, multiplication, and division) and begin work on fractions and decimals.

Success in mathematics depends on knowledge of content skills; moreover, research suggests that cognitive (e. g., planning, goal setting), metacognitive (e.g., calibration), and motivational (e. g., self-efficacy, intrinsic interest) variables also play a critical role (Zimmerman & Bandura, 1994; Schunk & Zimmerman, 2007). Elementary school students have difficulty with basic mathematics skills such as whole number operations, which present problems when they have to solve more complex fraction problems

(Labuhn, Zimmerman, & Hasselhorn, 2010; NMAP, 2008). Researchers suggest that learners can acquire new mathematical skills and develop self-regulatory competency through four sequential-levels of skill acquisition (Zimmerman & Kitsantas, 2002). The present study focuses on the first two levels of skill acquisition. Specifically, the research explores the role of cognitive modeling and social feedback on students' mathematics skills, self-efficacy, self-evaluation, and metacognitive judgments.

Social Cognitive Perspective of Self-Regulation

A social cognitive view on the development of self-regulatory competency postulates that novice learners acquire new skills and strategies through four sequential levels, (a) observational, (b) emulative, (c) self-controlled, and (d) self-regulated (Kitsantas & Zimmerman, 2000; Schunk & Zimmerman, 2007; Zimmerman, 2000). At the *observational level*, novice learners observe a model demonstrating and verbalizing the skills and strategies necessary to complete a specific task. *Cognitive modeling* occurs when models explain and demonstrate their thoughts and reasons while performing a cognitive task (Michembaum, 1977; Schunk, 1981). For example, cognitive modeling occurs when teachers demonstrate and explain various procedures while solving a fraction problem, which enhances students' learning of various skills and strategies. When problem-solving or using computation tools (e.g., strategy, calculator, or pencil-paper algorithm), teachers can model the choices they make and think aloud about them, and observing students can learn to make good choices (NCTM, 2000). Learners attain skill at this level when the observations facilitate a clear image of the processes necessary to solve fraction problems, for example recognizing another student's successful solution of a fraction problem. Learners become motivated at this level when they observe the model or another person attain success or receive praise after solving the problem. After observing the model, the next process is practicing the modeled behavior, which occurs at an emulative level.

At an *emulative level*, novice learners try to enact the model's performance. For example, after observing a teacher model a solution strategy with fraction problems, learners would understand the order of operations and would find the common denominator first before solving the problem. It should be noted that emulation is different from imitation. Imitation is merely copying every aspect of a model's behavior, whereas, emulation involves the abstraction and transfer of the underlying strategy to a new problem (Zimmerman, 2000). Guidance and feedback are essential during emulative experiences and can lead to higher levels of learning (Kitsantas & Zimmerman, 2000). The main source of motivation for learners at the emulative level is social feedback, often by the model regarding the student's work. *Feedback* is providing information to learners so they may

improve their skills, and it is related to higher achievement and superior motivation (Shute, 2008). Social feedback during emulative experiences enables learners to refine their skills and develop self-regulatory competency for further learning (Kitsantas & Zimmerman, 2000). These two levels are primarily social because observation and emulation occurs in the model's presence (Schunk & Zimmerman, 2007). Learning of the specific skill has begun but the process continues at the third and fourth levels where students require less assistance and gradually build skills for further learning.

At the *self-control* level, learners can use the skill or strategy independently when performing related tasks. For example, learners who have learned the order of operations in solving fraction problems would be able to use those skills to solve fraction problems with and without whole numbers. Students acquire competency at this level through independent practice. Learners attain this level proficiency when they compare their efforts with the standards obtained from the modeling experiences. Those who match or surpass those standards experience positive self-reaction, which is the primary source of motivation at this level (Kitsantas & Zimmerman, 2000). Researchers have measured self-reaction at this level by using a self-satisfaction scale (Zimmerman & Kitsantas, 2002; Ramdass & Zimmerman, 2011). However, this study will use a self-evaluation scale to measure self-reaction because students are evaluating their personal efforts with the standards obtained from the modeled experiences.

At the final level, *self-regulation*, learners are able to adapt their skills and strategies to various tasks, internal, and external conditions (Kitsantas & Zimmerman, 2000; Schunk & Zimmerman, 2007). For example, they would be able to solve fraction problems at a higher level of difficulty or decipher that a word problem involves fraction skills. In addition, learners self-monitor their performance and maintain their motivation through self-efficacy and intrinsic interest in completing the fraction problems. *Self-efficacy* is an individual's perceived capability of performing a task at designated levels (Bandura, 1997). Social influences are not present in these final two levels, but their influence never wanes completely and learners may seek help if a problem arises while completing a task (Zimmerman, 2000). Students who are able to use their skills to solve more difficult math problems would experience enhanced perceptions of self-efficacy, self-reactions, and intrinsic interest in this particular task and become more self-regulated.

Self-Efficacy

Self-efficacy is learners' perceived capabilities to learn and complete a task at designated levels (Bandura, 1997). Learners who have high self-efficacy for acquiring a skill or completing a task work harder and persist longer when they face difficulties compared to those who doubt their

capabilities (Bandura, 1997; Schunk & Zimmerman, 2007). Learners' develop self-efficacy through actual performances, modeled experiences, verbal persuasion, and physiological reactions (Bandura, 1986; 1997). Numerous studies indicate that mathematics self-efficacy influences the accuracy of mathematics performance, effort, and persistence (Hoffman & Schraw, 2009; Pajares, 1996; Ramdass & Zimmerman, 2008; Schunk & Ertmer, 2000). Experimental studies have shown that self-efficacy beliefs can be modified through effort and various self-regulatory processes, such as goal setting, modeling, self-monitoring, and self-evaluation of progress (Schunk & Ertmer, 2000; Schunk & Pajares, 2004). Although optimistic estimates of one's competency may increase effort and persistence initially, misjudgments of one's capabilities can be problematic if they lead to poor performance (Bandura, 1989).

Calibration and Achievement

Current research suggests that the accuracy of these self beliefs, calibration, is critical to academic success and motivation (Chen & Zimmerman, 2007; Klassen, 2006). *Calibration* is the accuracy of learners' perceptions of their performance (Huff & Nietfeld, 2009; Pieschl, 2009; Schunk & Pajares, 2004). Researchers suggest that calibration is one component in the process of developing self-regulatory competency and is a metacognitive skill for monitoring one's performance (Pieschl, 2009; Zimmerman, 2008). It refers to learners' awareness of what they know or do not know about their success as learners (Butler & Winne, 1995; Stone, 2000), which only captures a limited fraction of the entire realm of self-regulated learning (Pieschl, 2009). Decades of research has shown that students generally tend to overestimate their capabilities, which can have a detrimental impact on their learning and performance. Low-achieving students are less accurate and more overconfident than their high-achieving counterparts who tend to underestimate (Hacker & Bol, 2004; Klassen, 2006; Schunk & Pajares, 2004). Stone (2000) suggests that feedback plays an important role in influencing academic achievement and calibration.

Measuring Calibration

Social cognitive and metacognitive researchers have used various methods to measure calibration. Schraw, Potenza, and Nebelsick-Gullet (1993) used *mean bias*, which refers to the difference between estimated and true performance (the direction of judgment error). Bias measures of over- or underconfidence on a test or task are computed by taking the mean differences between predicted and actual performance scores and can range from -1 to 1. Scores larger than zero signify overconfidence, and scores less than zero correspond to underconfidence. *Mean accuracy* is a second measure,

and it assesses the magnitude of judgment error. *Mean accuracy* is the absolute value of the bias score. Accuracy can be calculated by squaring the bias score or taking its absolute value and can range from 0 to 1.

Social cognitive researchers (Chen, 2003; Pajares & Graham, 1999; Zimmerman, 2008) and metacognitive researchers (Huff & Neitfeld, 2009) have used the above two measures of bias and accuracy over the past decade to measure self-efficacy calibration. This study employs only the bias measure and it would be calculated using both self-efficacy and self-evaluation scores. Self-efficacy ratings are done before the task, whereas, self-evaluation ratings are done after the task. The rationale for including both is that students may display more accurate metacognitive monitoring after performing the task compared to before doing it.

Feedback

Feedback from another person can influence learning outcomes (Hattie & Timperley, 2007) because students become aware of which strategies were effective when completing tasks, which should enhance performance and calibration (Butler & Winne, 1995). Meta-analytic studies show that the role of feedback in classroom learning has been very effective with a mean effect size of .79 (Hattie & Timperley, 2007). Feedback occurs in the emulation phase in Zimmerman's (2000) model after students practice the modeled instructions on a new set of mathematics problems. Feedback at this stage is external because learners are still acquiring a skill, and it should enhance performance. Apart from improving performance, feedback may also enhance self-evaluative judgments, which occur after learners complete each problem. Recent experimental studies showed that feedback enhanced the accuracy of students' self-evaluative judgments compared to students who received no feedback (Labuhn, Zimmerman, & Hasselhorn, 2010) and modeling with social feedback improved students' math performance and metacognitive monitoring (Ramdass & Zimmerman, 2011). Feedback at the emulation level should enhance performance at the posttest level where students complete mathematics problems independently at the self-control level. One critical source of motivation at the self-control level is self-reaction. When students self-evaluate their work and find that they match or surpass the standards of the modeled experiences, they experience enhanced self-reactions, which motivate them to continue to acquire higher levels of mastery.

Self-Evaluation

Self-evaluation is judging one's performance based on a set of standards or objective outcomes (Zimmerman, 2000). For example, learners can evaluate their performance based on the standards they acquired from the modeling experiences. Self-evaluation enables learners to assess their

performance and develop reflective skills. Positive self-evaluations of one's capabilities and progress are important for maintaining self-efficacy for learning (Schunk, 2003). Self-evaluation has played an important role in enhancing students' academic achievement in both correlational and experimental studies (Schunk & Ertmer, 2000). Self-evaluation is a key self-regulatory process, which enhances self-efficacy for performance in the fourth level, self-regulation.

Research Evidence

Zimmerman and Kitsantas (2002) have examined the first two levels of observation and emulation in writing tasks with 72 college students and found that modeling improved students' writing skills, self-efficacy, self-satisfaction, and intrinsic interest. They assigned students randomly to six conditions. These conditions were three types of modeling (i. e., no modeling, mastery, and coping) and two types of feedback (i. e., feedback vs. no feedback). They hypothesized that students who observed a coping model would surpass those who observed a mastery model. Second, students who observed a mastery model would perform better than students who observed no model. The findings showed that a coping model improved students' writing skill and self-efficacy compared with the other two conditions, and the mastery condition improved the outcomes better than the no-model condition. In addition, social feedback was significant in improving students' writing skill compared to the no-feedback condition in the posttest (Zimmerman & Kitsantas, 2002). Specifically, they found support for the first two levels of the multilevel view of skill acquisition. Students' degree of observation learning significantly influenced learning in the emulative phase of practicing the writing skill.

In another study with 84 high school girls, Zimmerman and Kitsantas (1999) investigated the effects of goals on the students' writing skills. They assigned students randomly to six experimental conditions and one control group. These were goal setting (i.e., process goal, outcome goal, and shifting process-outcome goal) and self-recording (i. e., self-recording vs. no self-recording). The students observed a model demonstrate how to combine multiple sentences into one sentence by removing redundancies and adding transitional phrases. Students in the process goal condition concentrated on performing key steps in the task. Students in the outcome goal condition rewrote the sentences using the least number of words. Students in the shifting goal condition first focused on performing the key steps, but after a few minutes they shifted to the outcome goal of using a minimal amount of words. Half of the students in each condition recorded the number of strategy steps (i. e., process goal) they did correctly or the number of words (i.e., outcome goal) in the sentence. This study addressed the four processes of self-regulatory acquisition and competence. Modeling with practice

targeted the first two levels of observation and emulation. Process goals focused on the third level of self-control and outcome goals captured the fourth one, the self-regulated level.

The results indicate that students in the shifting goal condition evidenced higher self-efficacy and writing skill compared to the other conditions. In addition, students in the process goal were more effective than those in the outcome goal and no goal conditions. Students in the outcome goal group who skipped the third level of self-control had poorer writing skill and lower self-efficacy compared to those in the process and shifting goal conditions, implying that advancing through the stages may be beneficial in certain tasks (Schunk & Zimmerman, 2007; Zimmerman & Kitsantas, 1999).

Recently, Ramdass and Zimmerman (2011) examined the effects of coping and mastery modeling and social feedback on middle school students' algebraic skills. The researchers randomly assigned students to one of four conditions, (a) coping model with feedback, (b) coping model without feedback, (c) mastery model with feedback, and (d) mastery model without feedback. Students in the coping model condition observed a model making errors and later correcting them. Students in the mastery condition observed a model solving algebraic equations without making any errors. Feedback focused on the accuracy of students' responses. The results showed significant effects of modeling on students' algebraic skills, self-efficacy, and self-satisfaction, and calibration bias. Feedback was significant only on the calibration bias measure, implying that modeling and feedback not only improved students' algebraic skills and self-judgments, but also enhanced the accuracy of these self-judgments.

In an earlier study, Schunk (1981) evaluated the role of modeling on 56 elementary children arithmetic problem solving skills and self-efficacy judgments. The students were randomly to one of four treatment conditions, namely, (a) cognitive modeling with attribution, (b) cognitive modeling without attribution, (c) didactic instruction with attribution, (d) didactic without attribution, and e) control group.

The first hypothesis explored whether modeling, guided performance, and self-directed mastery would facilitate development of arithmetic skills and self-efficacy. The second related to the effects of effort attribution on achievement during arithmetic training. Finally, the third set of hypotheses tested the relationship of self-efficacy to subsequent achievement. Students received either modeling of division problems or didactic instruction and then practiced on a new set of problems. During the practice phase, half of the students in each treatment condition received effort attribution for success and difficulty.

The results showed that cognitive modeling was more effective in promoting mathematical skill compared to didactic instruction, but modeling-attribution condition had no effect on persistence. Students in the modeling condition showed significant improvement in self-efficacy, solved more

problems, and persisted longer compared to the control group. Effort attribution had no effect on self-efficacy or mathematics performance. Self-efficacy was not significant, but there was a relationship between self-efficacy and persistence; students who judged they could solve more problems, persisted longer on solving them. In addition, students in the modeling and attribution condition had the highest congruence between self-efficacy and math performance compared to the didactic and control conditions.

Present Study and Hypotheses

To date, apart from writing, research on this model of sequential skill acquisition in other subject areas such as mathematics or reading is lacking. Although Schunk's (1981) study focused on modeling, it was not based on Zimmerman's (2000) model for acquiring skill through sequential instruction. In addition, the issue of the accuracy of students' judgments was not fully explored. One primary goal of this study is to explore whether cognitive modeling would enhance fifth grade students' mathematical skills, self-efficacy, self-evaluation, and metacognitive skills. The first hypothesis states that students in the cognitive modeling condition would surpass those in the no-cognitive modeling conditions on the above outcomes. A second goal is evaluating the effects of social feedback during emulative learning. The second hypothesis states that students in the social feedback condition would surpass those who practice math problems without receiving feedback. As a result, the effects of cognitive modeling and social feedback were expected to enhance students' mathematics skill, self-efficacy, self-evaluation, and their metacognitive accuracy, which would provide support for the first two levels of sequential instruction of mathematics skills. In prior research with this model, Zimmerman and colleagues used a self-satisfaction scale to measure students' self-reactions after performance. In this study, a self-evaluation scale is used to evaluate students' self-judgments after the task. These goals are important for at least three reasons.

First, some researchers suggest that younger children have difficulty with metacognitive thinking, such as making a self-efficacy judgment (Berk, 2000); however, observing a model solve math problems students are currently studying should enhance their math skills, self-efficacy, self-evaluation, and metacognitive judgments. Students who observe an instructor model and verbalize how and why to use a strategy to solve a math problem can learn how to solve that problem (NCTM, 2000).

Second, the development of self-regulatory competency has been explored with students in higher grades. Research on younger children is lacking (Gaskill & Murphy, 2004). Self-regulation facilitates academic achievement (Zimmerman, 2008); therefore, it is important to study how younger children develop self-regulatory skills, which would shed more light on these processes from a developmental perspective.

Third, elementary students value and find mathematics important (NCTM, 2000), but in middle school their interest in mathematics, self-directedness, and intrinsic desire to learn begins to wane and this may affect mathematics performance (Fredericks, & Eccles, 2002; Pajares & Miller, 1994). Therefore, studying interventions that sustain younger children's motivation and improve mathematics performance would provide researchers and educators with tools to understand how to address issues of self-regulatory development and self-directedness in later grades.

Method

Participants

The participants were 88 (49 girls, 39 boys) English-speaking fifth grade students from three urban private schools. Students' mean age was 10 years 7 months ($SD = 6$ months). The ethnicity of the students was Caucasian (50%), South Asian Indian (22%), African American (14%), and Hispanic (14%). Participation in the study was voluntary, and required student assent and parental permission.

Task and Materials

The mathematics fraction problems were chosen from two math texts for elementary grades (Everyday Learning Corporation, 2004; Schwartz, 2008). The mathematical items were pilot tested on eight students, and a few problems were corrected to produce a range of problems from simple to difficult. The mathematics problems were presented in a stapled-worksheet containing 13 pages with ample space for students to display their work. In addition, students used pencils and erasers.

Measures

Mathematics skill. Each math problem was scored as correct 100 points, incorrect 0 points, or partial credit 50 points. Students received partial credit if they performed some steps toward solving the problem correctly, but did not complete the problem. The Cronbach's alpha for these items was .72. The predictive validity and construct validity of the math posttest is supported by the significant correlation among the other dependent measures. These math items were taken from math texts books and represented the work the students were doing on fractions at that time.

Self-efficacy. This task-specific scale assessed students' judgment of their capability during posttest before each of the six math problems (e.g., "How sure do you feel that you can solve this fraction problem correctly?"). It

was developed based on the guidelines of Bandura (2006). The scale ranged from 0 (not at all sure), 40 (somewhat sure), 70 (pretty sure) to 100 (very sure). The Cronbach's alpha for these six items was .90.

Self-evaluation. This scale assessed students' performance after solving each math problem during the posttest. Students were asked, "After solving the problem, how sure are you that you have solved it correctly?" It was adapted from Chen (2003) and it ranged from 0 (not at all sure), 40 (somewhat sure), 70 (pretty sure) to 100 (very sure). The Cronbach's alpha for these six items was .86.

Self-efficacy bias. Bias was calculated using procedures reported by Pajares and Graham (1999) and Schraw, Potenza, and Nebelsick-Gullet, (1993). Bias, the extent to which one is over- or underconfident in his/her judgement, is calculated at the item-level by subtracting each mathematics score from each self-efficacy score on the posttest (Chen, 2003). A student with a self-efficacy rating of 40 (somewhat sure) to solve the problem, but solved correctly (100) would have a bias score of -60 (40-100), indicating underconfidence. On the other hand, a student who rated his/her confidence to solve the problem as 70 (pretty sure) and solved it incorrectly (0) would receive a bias score of 70 (70-0), signifying overconfidence. As a result, self-efficacy bias scores could range from -100 to 100. The Cronbach's alpha for these six items was .65.

Self-evaluation bias. This measure was calculated in the above manner using the self-evaluation scores instead of self-efficacy scores. The Cronbach's alpha for these six items was .67.

Research Design

The researcher used a pretest-posttest group design. Students were randomly assigned to one of four conditions within each school, (1) cognitive modeling with social feedback, (2) cognitive modeling without social feedback, (3) social feedback without cognitive modeling, and (4) no cognitive modeling nor social feedback.

Procedure

The study was conducted with students individually in each school. All students returned the signed consent forms and provided demographic data (i.e., age, grade, gender, and ethnicity). Each session lasted roughly an hour and consisted of a pretest, a training phase, a practice phase, and a posttest. The researcher gave a brief overview of the study and told each student that participation was voluntary. Each student completed a set of six

fraction problems in the pretest phase, which lasted roughly 14 minutes and was identical for all students. Below is a description of the training phase according to each condition.

Cognitive modeling with social feedback. During the instruction phase students observed a model solve four fraction problems contained in the worksheet. The model demonstrated each step of the problem and verbalized aloud how to arrive at the correction solutions (see Appendix for sample problems). Training lasted for 14 minutes. In the practice phase, students solved four new math problems and received corrective feedback on each problem. If the solution was correct, the researcher told the student to proceed and solve the next problem. However, if the solution was incorrect, the researcher circled the error or errors and without giving the correct solution told the student, “This is the part you got wrong. What can you do to correct it?” This method of feedback was repeated for each of the four problems in the practice phase.

Cognitive modeling without social feedback. The treatment for this condition was similar to the one above except that in the practice phase students in this condition did not receive feedback. They were told to solve the four problems in the worksheet.

No cognitive modeling with social feedback. Students in this condition did not receive cognitive modeling instruction. After the pretest, they were advanced directly to the practice phase. After solving each problem, they received corrective feedback on their solutions. If the solution was correct, the researcher told the student to proceed and solve the next problem. However, if the solution was incorrect, the researcher circled the error or errors and without giving the correct solution told the student, “This is the part you got wrong. What can you do to correct it?” This feedback process was repeated for each of the four math problems in the practice phase.

No cognitive modeling or social feedback. In this condition, students did not receive cognitive modeling social or feedback. After the pretest, they were advanced directly to the practice phase where they solved the four problems, and then finally to the posttest.

The posttest phase occurred immediately after the practice phase. There were six problems in the posttest and each problem was shown briefly for students to make a self-efficacy judgment before solving it. After solving each problem, students rated their performance on the self-evaluation scale. This phase lasted roughly 20 minutes. For ethical reasons at the end of the study, students in the no-cognitive modeling with social feedback condition were taught cognitive modeling on problems in the training phase and students in the cognitive modeling condition without social feedback received

social feedback on the practice problems. Students in the no cognitive modeling and no social feedback condition were taught both cognitive modeling and got social feedback.

Results

The data were analyzed using multivariate analysis of covariance (MANCOVA) with pretest as the covariate, cognitive modeling and social feedback as independent measures, and posttest math, self-efficacy, self-evaluation, self-efficacy bias, and self-evaluation bias as dependent variables. This was followed by ANCOVA, and finally correlations between the dependent variables were calculated using Pearson correlation coefficient. To determine the effectiveness of random assignment, univariate analyses of variance of the pretest math fraction scores revealed no significant main effect or interaction effect, confirming that the groups were statistically comparable on math fraction skills before this study. Descriptive statistics are presented in Table 1.

Effects of Cognitive Modeling and Social Feedback on Practice Math

To assess the effects of cognitive modeling and social feedback on the practice mathematics data, ANCOVA results showed a main effect of cognitive modeling, $F(1, 83) = 58.21, p < .001$, partial $\eta^2 = .412$, implying that modeling at the observational level improved students' math performance significantly in the emulative phase compared to students in the no-cognitive modeling condition. However, feedback was not significant.

Table 1
Descriptive Statistics for Each Group

Measures	Modeling with Feedback <i>M (SD)</i>	Modeling No Feedback <i>M (SD)</i>	No Modeling with Feedback <i>M (SD)</i>	No Modeling No Feedback <i>M (SD)</i>
<i>Mathematics</i>				
Pretest	35.98 (26.28)	20.45 (24.36)	25.00 (26.35)	25.00 (22.27)
Practice	71.59 (24.45)	66.48 (26.55)	34.10 (29.17)	29.55 (21.67)
Posttest	72.73 (19.78)	68.56 (19.40)	46.59 (29.51)	36.36 (20.66)
Self-efficacy	65.61 (21.77)	56.44 (21.78)	47.88 (22.89)	52.27 (24.38)
Self-evaluation	69.92 (20.93)	58.79 (23.45)	47.95 (22.43)	46.44 (29.53)
<i>Calibration Bias</i>				
Self-efficacy	-7.12 (19.18)	-12.12 (17.31)	1.29 (27.02)	15.91 (25.57)
Self-evaluation	-2.80 (14.15)	-9.77 (12.15)	1.36 (26.06)	10.08 (24.88)

Effects of Cognitive Modeling and Social Feedback on Posttest Measures

MANCOVA results showed a main effect of cognitive modeling, Wilk's $\lambda = .58$, $F(3, 81) = 19.81$, $p < .001$, partial $\eta^2 = .423$, and a weak interaction of cognitive modeling and feedback, Wilk's $\lambda = .92$, $F(3, 81) = 2.26$, $p < .088$, partial $\eta^2 = .077$.

Subsequent univariate analysis of covariance (ANCOVA) showed a main effect of cognitive modeling for each of the dependent variables. Math fraction skills, $F(1, 83) = 60.63$, $p < .001$, partial $\eta^2 = .42$. There was no main effect of social feedback. However, a significant interaction effect of cognitive modeling and social feedback, $F(1, 83) = 5.18$, $p < .025$, partial $\eta^2 = .059$, indicates that a combination of both variables improved students' math performance significantly.

On *self-efficacy*, $F(1, 83) = 5.10$, $p < .027$, partial $\eta^2 = .058$, and *self-evaluation* $F(1, 83) = 12.39$, $p < .001$, partial $\eta^2 = .13$. There was no main effect of social feedback or interaction effect on these measures.

On the calibration bias measures, there was a main effect of cognitive modeling on both variables. On *self-efficacy bias*, $F(1, 83) = 13.65$, $p < .001$, partial $\eta^2 = .14$, and a significant interaction effect, $F(1, 83) = 5.25$, $p < .024$, partial $\eta^2 = .059$.

On *self-evaluation bias*, $F(1, 83) = 7.27$, $p < .001$, partial $\eta^2 = .081$, and a significant interaction effect, $F(1, 83) = 3.98$, $p < .049$, partial $\eta^2 = .046$. There was no main effect of feedback on both calibration measures.

Figure 1 shows significant interaction effects on self-efficacy bias, indicating that cognitive modeling and social feedback improved the accuracy judgments of students compared to students in the no modeling and no feedback conditions.

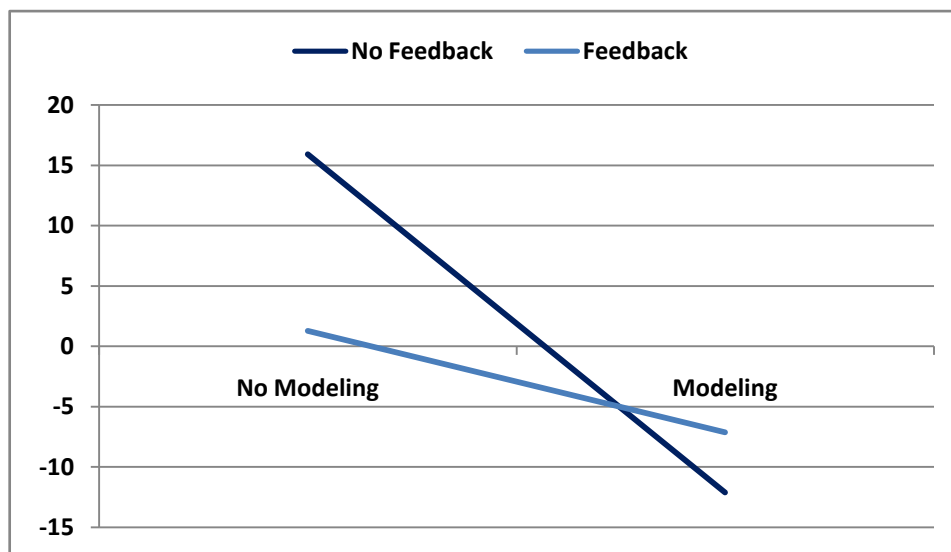


Figure 1. Interaction Effects on Self-Efficacy Bias

Correlation Analysis

Correlation among the variables is presented in Table 2. All the variables correlated positively and significantly with posttest math, except the calibration measures. The negative correlations between the calibration measures (i.e., self-efficacy bias, $r = -.60$; self-evaluation bias, $r = -.45$) and posttest math indicate that overconfidence diminished students' math performance. Self-efficacy and self-evaluation correlated very high ($r = .85$), which extends a prior finding by Chen (2003). In addition, the two calibration measures also correlated positively ($r = .84$), implying that they both measured students' metacognitive monitoring.

Table 2
Bivariate Intercorrelations among Dependent Measures

Measures	1	2	3	4	5	6	7
Math Pretest	----						
Math Practice	.47**	----					
Math Posttest	.61**	.71**	----				
Self-efficacy	.53**	.42**	.53**	-----			
Self-evaluation	.55**	.47**	.86**	.85**	----		
Self-efficacy Bias	-.17	-.39**	-.60**	.37**	-.07	----	
Self-evaluation Bias	-.12	-.34**	-.45**	.37**	.35**	.84**	----
Mean	26.61	50.43	56.06	55.55	55.78	-0.51	-0.28
Standard Deviation	25.11	31.45	27.00	23.29	25.71	24.68	21.20
Cronbach's Alpha	.75	.75	.72	.90	.86	.65	.67

Note. ** $p < .01$

Discussion

Using a social cognitive model of self-regulatory development, the present study explored whether cognitive modeling and social feedback would improve fifth grade students' mathematics skills, self-efficacy, self-evaluation, and calibration (i.e., the accuracy of these judgments). The results revealed a significant main effect of cognitive modeling on each dependent measure, implying that cognitive modeling led to superior math performance, enhanced self-efficacy, self-evaluation, and metacognitive monitoring compared to the no- cognitive modeling conditions. These findings provide support for the first hypothesis, which postulated a main effect of cognitive modeling.

Although students begin learning fraction skills in fourth grade, most students experience difficulty in this area. This study shows that modeling of strategies and skills to solve fraction problems enhanced students' learning and skill acquisition. In a prior study, Schunk (1981) found that cognitive modeling improved the arithmetic skills and self-efficacy of children with low

arithmetic achievement. The sample in this study was normal achieving students and the results show that modeling exerted a significant influence in their learning of fraction skills, which was evident from both practice and posttest mathematics scores. During emulative practice, there was also a main effect of cognitive modeling on practice math, and practice math scores correlated significantly with posttest math scores ($r = .71$), implying that observing a model significantly improved the effectiveness of practice at the emulative phase. Significant effects of cognitive modeling on posttest mathematics imply that a combination of observational and emulative effects enabled students to solve the problems, signifying that they achieved some success in the third level of self-control. Moreover, significant self-evaluation findings confirm that students in the cognitive modeling condition experienced positive self-reactions compared to those in the no-cognitive modeling condition.

The second objective was whether social feedback would improve math performance, self-efficacy, self-evaluation, and calibration judgments. MANCOVA results showed marginal significance at the $p < .10$ level. Follow-up ANCOVA analysis showed no main effect of social feedback; however, there were significant interaction effects on posttest math and both calibration measures, implying that a combination of cognitive modeling and social feedback improved fifth grade students' mathematics performance and also their metacognitive monitoring. This is an important finding because research on elementary students' metacognitive skills is lacking. This study shows that a training protocol was effective in improving these students' mathematics performance and calibration judgments. Although practice time was short to acquire proficiency in the fraction skill, the results indicate that practice is essential in acquiring knowledge of core arithmetical concepts.

In this study, feedback focused on performance, and it was corrective and highlighted errors. Feedback is more effective for novice learners when it is explanatory and provides details on how to improve task performance rather than indicating the accuracy of the answer or highlighting errors (Shute, 2008). Nevertheless, significant interaction effects on the math posttest, self-efficacy bias, and self-evaluation bias suggest that a combination of cognitive modeling and social feedback contributed to better math performance and lower self-efficacy and self-evaluation bias. This latter finding was an interesting one as it suggests that there was no difference in students' awareness of what they knew before solving the problem (i.e., self-efficacy bias) or after solving it (i.e., self-evaluation bias). It implies that students in the cognitive modeling condition were confident in their metacognitive skills from the beginning of the task to the end. Finally, this study adds to the body of research on the social cognitive model of sequential skill acquisition that learners can acquire new mathematical skills through observation, emulation, self-control, and self-regulation (Ramdass & Zimmerman, 2011; Zimmerman & Kitsantas, 1999; 2002).

Educational Implications

The implications of this study are manifold. First, it shows that fraction skills can be taught in fifth grade, which is a recommendation by NCTM (2000). Classroom efforts should be dedicated at mastery of mathematical skills and the focus should not be solely on students' successful performance on standardized tests. Second, cognitive modeling of strategies, verbalizing, and explaining the steps in the solution process are critical for students' comprehension of novel concepts. In addition, students need ample time to practice new math skills until they acquire proficiency. Third, feedback on students' work is important in enabling them to learn what they did correctly and improve on their mistakes. This also facilitates metacognitive awareness, which is a key component of self-regulatory development. Fourth, math tasks varied in this study from easy to difficult. It is important to have a range of tasks so students can develop competency to master more difficult problems. According to NMAP (2008), in the United States, elementary textbooks have easier arithmetic problems more often than difficult problems, whereas the opposite occurs in countries with higher mathematics achievement, such as Singapore and Hong Kong.

Limitations and Future Research

Like all research studies the present results should be interpreted with caution. First, students were tested individually and therefore, the findings may not be transferable to classroom settings. Future studies to test this model should be conducted with groups of students in classroom to determine how to promote self-regulated learning in actual classrooms.

Second, one weakness of this study is the type of feedback students' received. Students received corrective feedback if their responses were correct. However, if it was incorrect, the researcher highlighted the errors without any further information. As a result, feedback exerted a weak effect on students' math performance, and self-judgments. From a self-regulatory perspective, more elaborate feedback would enable students to learn how to correct their errors, which is important for skill development and metacognitive monitoring.

It is not clear how students develop internal feedback at the third level. The results seem to suggest that students in the cognitive modeling condition generated internal feedback, which enabled successful performance in the posttest phase. Instead of giving students feedback, it would be better to discover how students adjust their performance to the standards they derive from the modeled experiences. This type of feedback would be critical to success for transfer tasks, which should be explored in future studies. One may speculate that self-regulatory competency in a specific task would facilitate transfer of skills in solving a novel set of problems.

Conclusion

In conclusion, this study supports the social cognitive perspective of self-regulation that learning novel mathematics skills requires social guidance initially until students develop proficiency and self-efficacy in their capability. During observation, exposure to a model before practicing is critical for skill development and feedback is important during practice as it conveys important information that allows learners to gauge their skill development and monitoring their work.

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Appendix

Examples of Fraction Problems

1. From a $9 \frac{4}{48}$ gallon container of milk, Sara used $6 \frac{11}{12}$ gallons to make a cake. How much milk remained?
2. From $\frac{8}{17}$ yard length of cloth, $\frac{37}{85}$ yard was cut. How much yard remained?
3. In a math test, the three top students finished in $9 \frac{3}{4}$ minutes, $9 \frac{5}{6}$ minutes, and $9 \frac{7}{9}$ minutes. How much minutes separated the first and third student?
4. A box had $\frac{72}{81}$ pound of cookies. For breakfast, some children ate $\frac{60}{72}$ pound of cookies. How much pound of cookies remained?

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About the Author

Darshanand Ramdass is a postdoctoral fellow at the City University of New York. Currently his research is at New York University, Create Lab where he is involved with a team of researchers in developing and testing educational computer games to facilitate mathematics learning in middle school children. His research interests are social cognitive theory of learning and motivation, self-efficacy, calibration, attention and memory, and yoga. His email address is: dramdass@gc.cuny.edu